STEM Innovation Academy Unit 1

Subject: Integrated Math III Unit Title: Polynomial functions and equations Grade: 10	Teacher: Noah Singman Duration: 8 weeks

Summary of Unit

The purpose of this unit is to have a transition phase from Integrated Math II (geometry) course to Integrated Math III (Algebra II) course. The lessons and time will be planned based on procedural learning, and some conceptual learning of quadratic equations, graphing, and formulas that the students have encountered in Integrated Math I (Algebra I) course. There are three distinct related topics within this unit;
1) Quadratic equations and complex numbers;
2) Key characteristics of and sketching polynomials as well as solving Cubic equations;
3) Remainder theorem and long division of polynomials.

Stage 1 – Desired Results

Essential Questions:

- What is polynomial function?
- How do you perform arithmetic operation on polynomials?
- How do you interpret key features of graphs and tables in terms of the quantities?
- How do you describe concavity of a graph?
- How do you identify odd and even function based on the symmetry?
- What is a rational expression?
- How do you simplify rational expressions?
- How do you re-write rational expressions?
- How are the degrees of polynomials related to its' zeroes?
- How can you analyze functions using different representation?
- How do you sketch graphs showing key features given a verbal description of the relationship?
- What is the difference between absolute values and relative values?
- What is a short-term behavior?
- What is a long-term behavior?
- How can you analyze functions using different representation?
- What is polynomial equation?
- What is a complex number?
- How do you solve polynomial equation?
- How does discriminant help you make prediction about roots of quadratic equations?
- What is synthetic division?
- What is the fundamental theorem of Algebra?
- What is the remainder theorem?

Objectives:

- Given a quadratic graph and its function in standard form, students will identify key features of the graph, and justify zeros algebraically.
- Given a quadratic function in standard students will sketch the graph showing y-intercept & end behavior.
- Identify key features of quadratic functions in factored form and standard form and sketch showing key features.

- Using a graphing calculator to graph a quadratic function and use the graph to re-write the standard form into factored form.
- Re-write standard form in to factored form using area model, or any factoring strategies.
- Solve quadratic equations in standard form by factoring or graphing and sketch the graph to show key features.
- Given a vertex form of the quadratic function students will identify key features of the vertex and sketch the graph of the quadratic equation in vertex form without the calculator.
- Given a vertex form of the quadratic function students will solve simple quadratic equations and sketch the graph to show key features and solve quadratic equations in vertex form by taking square roots.
- Solve quadratic equations by completing squares.
- Derive the quadratic formula.
- Apply the quadratic formula to find real solutions
- Identify the nature of the roots and number of real roots from graphs and the discriminant.
- By using the quadratic formula and the definition of imaginary numbers students will solve and graph Quadratic equations with non-real solutions and derive the definition of complex numbers.
- By using the definition of complex number students will simplify complex numbers and perform operations with complex numbers.
- Given a polynomial function in factored form students will identify zeros and y-intercept; plot them on the coordinate plane; develop strategies to find the end behavior; create a sketch of the cubic function; identify the end behavior of functions with positive and negative leading coefficients
- Given a real-life situation students will create a mathematical model (polynomial function) to represent the situation; sketch the graph; identify key features from graphs and equations; and interpret the key features in terms of context
- Understand the concept of long division
- Perform long division on polynomial
- Rewrite simple rational expression in different form
- Through long division and by evaluating the polynomial for a given root students will understand the remainder theorem and apply the remainder theorem to find the remainder of a polynomial and see the connection between factor and the remainder.
- Re-write the sum and difference of cube as factored form
- Using area model or GCF students will factor cubic polynomials.
- By performing factoring by grouping or long division and quadratic formula students will solve cubic equations; identify key features; create a rough sketch and show key features of the polynomial function.

Standards/Outcomes: NJSLS

- **A.APR.2** – Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on divisions by $x - a$ is $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
- **A.APR.6** – Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **A.SSE.2** – Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.
- **A.SSE.3** – Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

actor a quadratic expression to reveal the zeros of the function it defines.
omplete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

se the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $\left(1.15^{\frac{1}{12}}\right)^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

- **A.REI.4** – Solve quadratic equations in one variable.

se the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

olve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

- **A.REI.11** – Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- **F.IF.4** – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- **F.IF.6** – Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **F.IF.7** – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

raph linear and quadratic functions and show intercepts, maxima, and minima.

raph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

raph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

raph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- **F.IF.8a** – Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

se the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- **N.CN.1** – Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- **N.CN.2** – Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Unit Math Practice Standards:

- **MP.1** Make sense of problems and persevere in solving them.
- **MP.2** Reason abstractly and quantitatively.
- **MP.3** Construct viable arguments and critique the reasoning of others.
- **MP.4** Model with mathematics.
- **MP.5** Use appropriate tools strategically.
- **MP.6** Attend to precision.
- **MP.7** Look for and make use of structure.
- **MP.8** Look for and express regularity in repeated reasoning.

<table>
<tr><th colspan="2">Stage 2 – Assessment Evidence</th></tr>
<tr><td>Performance Task(s):
Performance Task 1: Polynomial Equation Proof
Performance Task 3: Box Volume
Performance Task 4: Graphing from Roots
Authentic Experiences:</td><td>Unit Pre-Assessment:
Unit 1 Readiness Assessment
NWEA Diagnostic Assessment
Benchmark 1 Assessment
Presentation:</td></tr>
<tr><td>Extensions (Tier I):
Practice packets
Visual Presentations/Lessons
More applications of findings</td><td>Differentiation (Tiers 2 and 3):
Selective grouping
Extended time
Small groups / Individual instruction</td></tr>
<tr><th colspan="2">Stage 3 – Learning Plan</th></tr>
<tr><td colspan="2">Vocabulary
Standard form: ax^2+bx+c
Leading Coefficient
Constant
Factors
FOIL
Greatest Common Factor
Roots
X-intercepts
Solutions
Completing the Square
Quadratic Formula
Square Root/Plus and minus
Vertex
Axis of Symmetry
Domain/Range</td></tr>
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